**Proof 1**: **SrCWMTPr(A) ≥ SrPr(A)**

We want to know for what values of p and q: (1) SrCWMTPr(A) ≥ SrPr(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

SrCWMTPr(A) = p\*(-p\*\*3\*q + 4\*p\*\*2\*q\*\*2 - 3\*p\*\*2\*q + p\*\*2 - p\*q\*\*3 - 5\*p\*q\*\*2 + 9\*p\*q - 3\*p + q\*\*3 + q\*\*2 - 5\*q + 3)

And

SrPr(A) = p\*(p\*\*2 + 3\*p\*q\*(p - 1)\*(q - 1) + 3\*p\*(p - 1)\*(q - 1) + 3\*(p - 1)\*\*2\*(q - 1)\*\*2)

Then we can rearrange (1) to produce SrCWMTPr(A) – SrPr(A) ≥ 0 which becomes:

(p\*(-p\*\*3\*q + 4\*p\*\*2\*q\*\*2 - 3\*p\*\*2\*q + p\*\*2 - p\*q\*\*3 - 5\*p\*q\*\*2 + 9\*p\*q - 3\*p + q\*\*3 + q\*\*2 - 5\*q + 3)) – (p\*(p\*\*2 + 3\*p\*q\*(p - 1)\*(q - 1) + 3\*p\*(p - 1)\*(q - 1) + 3\*(p - 1)\*\*2\*(q - 1)\*\*2)) ≥ 0

When we simplify this, we arrive at the following inequality:

p\*q\*(-p\*\*3 - 2\*p\*\*2\*q + 3\*p\*\*2 - p\*q\*\*2 + 4\*p\*q - 3\*p + q\*\*2 - 2\*q + 1) ≥ 0

This can be factored and simplified again to give the following:

p\*q\*(1-p)\*(p+q-1)\*\*2 ≥ 0

From this we get the solution 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1, meaning that the inequality SrCWMTPr(A) ≥ SrPr(A) is true for all values of p and q. We can also see that SrCWMTPr(A) = SrPr(A) only when p+q=1, p=0, or q=0, p=1.

**Proof 2**: **SrCWMTPr(A) ≥ CrPr(A)**

We want to know for what values of p and q: (1) SrCWMTPr(A) ≥ CrPr(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

SrCWMTPr(A) = p\*(-p\*\*3\*q + 4\*p\*\*2\*q\*\*2 - 3\*p\*\*2\*q + p\*\*2 - p\*q\*\*3 - 5\*p\*q\*\*2 + 9\*p\*q - 3\*p + q\*\*3 + q\*\*2 - 5\*q + 3)

And

CrPr(A) = p\*(6\*p\*\*2\*q\*\*2 - 6\*p\*\*2\*q + p\*\*2 - 9\*p\*q\*\*2 + 12\*p\*q - 3\*p + 3\*q\*\*2 - 6\*q + 3)

Then we can rearrange (1) to produce SrCWMTPr(A) – CrPr(A) ≥ 0 which becomes:

(p\*(-p\*\*3\*q + 4\*p\*\*2\*q\*\*2 - 3\*p\*\*2\*q + p\*\*2 - p\*q\*\*3 - 5\*p\*q\*\*2 + 9\*p\*q - 3\*p + q\*\*3 + q\*\*2 - 5\*q + 3)) – (p\*(6\*p\*\*2\*q\*\*2 - 6\*p\*\*2\*q + p\*\*2 - 9\*p\*q\*\*2 + 12\*p\*q - 3\*p + 3\*q\*\*2 - 6\*q + 3)) ≥ 0

When we simplify this, we arrive at the following inequality:

p\*q\*(-p\*\*3 - 2\*p\*\*2\*q + 3\*p\*\*2 - p\*q\*\*2 + 4\*p\*q - 3\*p + q\*\*2 - 2\*q + 1) ≥ 0

This can be factored and simplified again to give the following:

p\*q\*(1-p)\*(p+q-1)\*\*2 ≥ 0

From this we get the solution 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1, meaning that the inequality SrCWMTPr(A) ≥ CrPr(A) is true for all values of p and q. We can also see that SrCWMTPr(A) = CrPr(A) only when p+q=1, p=0, or q=0, p=1.

**Proof 3**: **SrCWMTPr(A) ≥ TrPr(A)**

We want to know for what values of p and q: (1) SrCWMTPr(A) ≥ TrPr(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

SrCWMTPr(A) = p\*(-p\*\*3\*q + 4\*p\*\*2\*q\*\*2 - 3\*p\*\*2\*q + p\*\*2 - p\*q\*\*3 - 5\*p\*q\*\*2 + 9\*p\*q - 3\*p + q\*\*3 + q\*\*2 - 5\*q + 3)

And

TrPr(A) = p\*(-p\*\*3\*q + p\*\*3 + 4\*p\*\*2\*q\*\*2 - p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - 4\*p\*q\*\*2 + 5\*p\*q + q\*\*3 - 3\*q + 2)

Then we can rearrange (1) to produce SrCWMTPr(A) – TrPr(A) ≥ 0 which becomes:

(p\*(-p\*\*3\*q + 4\*p\*\*2\*q\*\*2 - 3\*p\*\*2\*q + p\*\*2 - p\*q\*\*3 - 5\*p\*q\*\*2 + 9\*p\*q - 3\*p + q\*\*3 + q\*\*2 - 5\*q + 3)) – (p\*(-p\*\*3\*q + p\*\*3 + 4\*p\*\*2\*q\*\*2 - p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - 4\*p\*q\*\*2 + 5\*p\*q + q\*\*3 - 3\*q + 2)) ≥ 0

When we simplify this, we arrive at the following inequality:

p\*(-p\*\*3 - 2\*p\*\*2\*q + 3\*p\*\*2 - p\*q\*\*2 + 4\*p\*q - 3\*p + q\*\*2 - 2\*q + 1) ≥ 0

This can be factored and simplified again to give the following:

p\*(1-p)\*(p+q-1)\*\*2 ≥ 0

From this we get the solution 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1, meaning that the inequality SrCWMTPr(A) ≥ TrPr(A) is true for all values of p and q. We can also see that SrCWMTPr(A) = TrPr(A) only when p+q=1, p=0, p=1.

**Proof 4**: **SrPr(A) = CrPr(A) ≥ TrPr(A)**

Since SrPr(A) = CrPr(A) , let us try to prove for what values of p and q: (1) SrPr(A) ≥ TrPr(A) which would prove CrPr(A) ≥ TrPr(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

SrPr(A) = p\*(p\*\*2 + 3\*p\*q\*(p - 1)\*(q - 1) + 3\*p\*(p - 1)\*(q - 1) + 3\*(p - 1)\*\*2\*(q - 1)\*\*2)

And

TrPr(A) = p\*(-p\*\*3\*q + p\*\*3 + 4\*p\*\*2\*q\*\*2 - p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - 4\*p\*q\*\*2 + 5\*p\*q + q\*\*3 - 3\*q + 2)

Then we can rearrange (1) to produce CrPr(A) – TrPr(A) ≥ 0 which becomes:

(p\*(p\*\*2 + 3\*p\*q\*(p - 1)\*(q - 1) + 3\*p\*(p - 1)\*(q - 1) + 3\*(p - 1)\*\*2\*(q - 1)\*\*2)) – (p\*(-p\*\*3\*q + p\*\*3 + 4\*p\*\*2\*q\*\*2 - p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - 4\*p\*q\*\*2 + 5\*p\*q + q\*\*3 - 3\*q + 2)) ≥ 0

When we simplify this, we arrive at the following inequality:

p\*(p\*\*3\*q - p\*\*3 + 2\*p\*\*2\*q\*\*2 - 5\*p\*\*2\*q + 3\*p\*\*2 + p\*q\*\*3 - 5\*p\*q\*\*2 + 7\*p\*q - 3\*p - q\*\*3 + 3\*q\*\*2 - 3\*q + 1) ≥ 0

This can be factored and simplified again to give the following:

(1-p)\*(1-q)\*p\*(p+q-1)\*\*2 ≥ 0

From this we get the solution 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1, meaning that the inequality SrPr(A) = CrPr(A) ≥ TrPr(A) is true for all values of p and q. We can also see that SrPr(A) = CrPr(A) = TrPr(A) only when p+q=1, p=0, p=1, q=1.